



## A NEW APPROACH FOR MATHEMATICAL MODELING OF DARK CURRENT CHARACTERISTICS OF QUANTUM WIRE INFRARED PHOTODETECTOR

H. BARICKABY

Department of communications and Electronics  
Shiraz University  
Shiraz, Iran  
[h.barickaby@yahoo.com](mailto:h.barickaby@yahoo.com)

A. ZARIFKAR, M. H. SHEIKHI

Department of communications and Electronics  
Shiraz University  
Shiraz, Iran  
[zarifkar@shirazu.ac.ir](mailto:zarifkar@shirazu.ac.ir), [msheikhi@shirazu.ac.ir](mailto:msheikhi@shirazu.ac.ir)

Received 19/02/2011, accepted 9/03/2011, online 29/03/2011

**Abstract**-In order to study the dark current characteristics in a quantum wire infrared photodetector (QRIP), one must get the average number of electrons in quantum wires (QRs), which is mostly too complicated. In this paper we give a simple formula to calculate the average number of carriers in a quantum wire (QR) that can be easily evaluated by mathematical softwares, and then we will use this formula to study dark current characteristics of a quantum wire infrared photodetector (QRIP).

**Keywords:** *Quantum wire infrared photodetectors, quantum wire, dark current ,mathematical formulation,modeling.*

### I. INTRODUCTION

Quantum wire infrared photodetectors (QRIPs) have been under focusing consideration among the new nano-sized intersubband photodetectors, mostly because of their low dark current [1], wide spectral range, ability of absorbing the normal incident light [2], and high bit rate data transfer [3] to name a few. Confinement of electrons in one dimensional quantum wires will lead to discrete bound energy levels so that capture probability and relaxation time of electrons will increase [2]. These phenomena eventually give lower dark current, more enhanced responsivity, and better signal-to-noise ratio. The material of their vertical-based nano heterostructure have been under different examinations [2, 3]. Among some qualifying materials GaAs, AlGaAs, and InGaAs have been chosen [1, 3]. One of the most important parameters of QRIPs is the dark current, which flows even without the presence of the incident light, and so takes a major part in determining of noise in QRIPs . The major contribution to dark current formula in QRIP was carried out by V.Ryzhii [4, 5].

Different treatments are applied to model the characteristics of QRIPs [1-3]. In this paper we will utilize a new simple mathematical formulation to consider the characteristics of a QRIP under dark current condition. In section 2, we will have a brief review on basic concepts of dark current physical origin. Section 3 discusses the new mathematical modeling to reach the final dark current equation in a more simple formula. And in our last section there will shown some figures that prove our results to be in high agreement with the referenced ones.

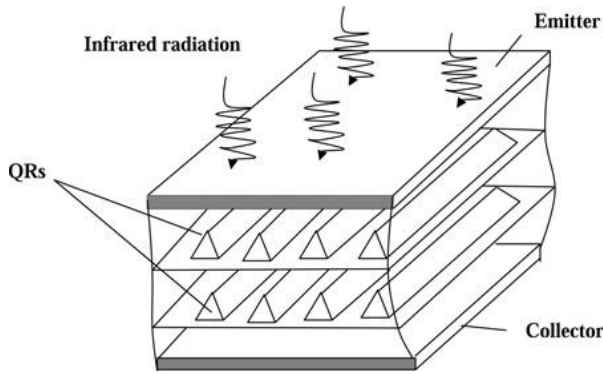
### II. DARK CURRENT BASIC CONCEPTS

Under dark current condition, the space charge in the active region of a QRIP changes due to two counteracting processes:

- 1) Electron capture into Quantum wires;
- 2) Thermo emission of electrons from them.

Then the dominant source of dark current in QRIPs is of the thermionic origin. Let's consider the simplified structure of a QRIP shown in Fig. 1. Under no light exposure, we assume that there are  $\{k = 1,2,\dots,M\}$  layers of QRs arrays, each layer containing,  $\langle Nk \rangle$ , average number of electrons. In the active region of a QRIP the drift current due to electrons captured and excited from QRs is governed by [1]:

$$\langle j_d \rangle = q \sum QR G_k / P_k \quad (1)$$



**Fig.1** Simplified structure of a QRIP

In which,  $j_d$  is the dark current, density ( $q$ ) is the electron charge,  $G_K$  is the electron thermo – excitation from QRs, and  $P_K$  is the probability of electron capture. For voltages satisfying ( $qV > qV_0 \gg KT$ ), the mobile carriers injected from the emitter contact, reach the first QRs layer plane and then overcome the barrier height ( $qV_0 = E_{QR}$ ). Now the dark current formula takes this from [1]:

$$j_d = j_m \exp\left(\frac{q(\langle\varphi\rangle + \varphi)}{KT}\right) \quad (2)$$

Where ,  $j_m$ ,  $\varphi$ ,  $K$ ,  $T$  represents maximum current density being injected from emitter, potential in the QR base plane, where symbol,  $\langle \dots \rangle$ , means averaging over the base plane, Boltzman constant and temperature.

### III. MATHEMATICAL MODELING

To calculate the average potential we begin with the Poisson’s equation [2]:

$$\frac{d^2 \langle \varphi \rangle}{dz^2} = \frac{4\pi q}{\epsilon} \sum_{k=1}^M (\langle N \rangle \sum_{QR} - \sum D) \quad (3)$$

And the potential in the QR base plane is given by [1]:

$$\varphi = \frac{2\sqrt{2}}{\epsilon} \langle N \rangle q \left( 0.36 - \frac{1}{2} \sum QR (x^2 + y^2) \right) \quad (4)$$

By numerical solution of (3) and two boundary conditions:

- 1)  $\varphi|_{z=0} = 0$  (At the emitter) and
- 2)  $\varphi|_{z=(M+1)L} = V$  (At the collector)

We get [1]:

$$\langle \varphi \rangle = \frac{V}{M+1} - \frac{2\pi q M L}{\epsilon} (\langle N \rangle \sum QR - \sum D) \quad (5)$$

The capture probability in a QR has this form [1]:

$$C = P_k \left( \frac{N_{QR} - \langle Nk \rangle}{N_{QR}} \right) \exp\left( \frac{-q^2 \langle NK \rangle}{CKT} \right) \quad (6)$$

In which,  $C = 0.36\sqrt{a_{QR}}$  is the capacitance of a QR and  $\epsilon$  being the matter permittivity.

Now we solve (6) to find,  $\langle NK \rangle$ . As  $\langle NK \rangle$  is same for all indices (i.e.  $\langle NK \rangle = \langle N \rangle$ ), then we simply use  $\langle N \rangle$ .

Assuming:  $A_1 = \frac{q^2}{CKT}$ ,  $A_2 = N_{QR} \frac{P_C}{P_0}$ ,

$A_1 \langle N \rangle = x$  ,  $A_1 A_2 = A_3$  , we have:

$$A_3 = A_1 (N_{QR} - \langle N \rangle) e^{-x} \quad (7)$$

$$A_3 e^x = A_1 N_{QR} - x \quad (8)$$

Then using the Maclaurin series of ( $e^x$ ) up to third harmonics, we get:

$$A_3 \left( 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \right) = A_1 N_{QR} - x \quad (9)$$

Now we can easily solve the cubic equation using following formula:

$$x = u - \frac{p}{3u} - \frac{a}{3}, \langle N \rangle = \frac{X}{A_1} \quad (10)$$

that,

$$P = 6\left(1 + \frac{1}{A_3}\right) - 3,$$

$$q = 6\left(1 - \frac{A_1}{A_3} N_{QR}\right) - 6\left(1 + \frac{1}{A_3}\right) + 2$$

$$U = \left[ -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{P^3}{27}} \right]^{1/3}$$

Integration  $\langle j_d \rangle$  over the in-plane coordinates we get the average current density as [1]:

$$\langle j_d \rangle = \sum_{QR} \int_{\frac{-l_{qr}}{2}}^{\frac{l_{qr}}{2}} \int_{\frac{-L_{QR}}{2}}^{\frac{L_{QR}}{2}} j_d \, dx dy \quad (11)$$

In which  $L_{QR}$  in the longitudinal length size of QRs. Solving (11) by numerical methods, we have:

$$\langle j_d \rangle = j_m \frac{\theta}{\langle N \rangle} \left( \frac{0.1q(V + C_1 - \langle N \rangle C_2)}{(M + 1)KT} \right) \quad (12)$$

While  $\theta$  is the non-uniformity parameter appearing as:

$$\theta = 21.45 \operatorname{erf} \left( 0.48ql_{qr} \sqrt{\frac{\langle N \rangle \sqrt[3]{\sum QR}}{\epsilon KT}} \right) \frac{\epsilon KT}{q^2 \sqrt{\sum QR}} \quad (13)$$

$$C_1 = \frac{2\pi q}{\epsilon} M(M + 1)L \sum D$$

$$C_2 = \frac{2\pi q}{\epsilon} M(M + 1) \sum QRL(1 - 0.32/ML \sqrt{\sum QR})$$

In which  $\sum_{QR}$ ,  $\sum_D$ ,  $L$  stands for the density of QRs arrays, the donor density of QRs arrays and the transverse spacing between QRs respectively.

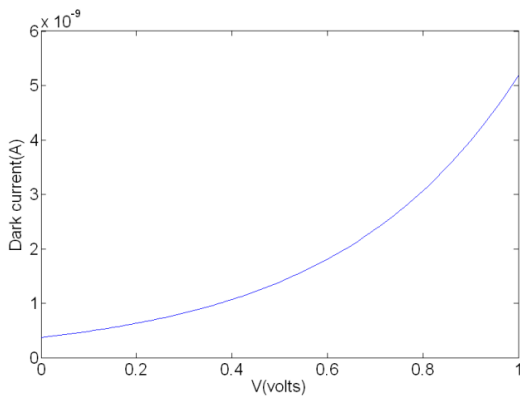
Now substituting the value of  $\langle N \rangle$  from (10) and  $\theta$  in (11) the final equation for dark current will be

$$\langle j_d \rangle = \frac{21.45}{x} \operatorname{erf} \left( 0.48ql_{qr} \sqrt{\frac{x \sqrt[3]{\sum QR}}{\epsilon KT}} \right) \frac{\epsilon KT}{q^2 \sqrt{\sum QR}} \quad (14)$$

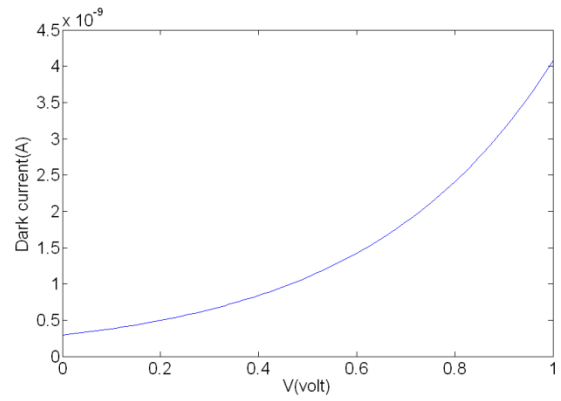
$$(q(v + C_1 - xC_2)/(M + 1)KT)$$

## VI. NUMERICAL RESULTS AND SUMMARY

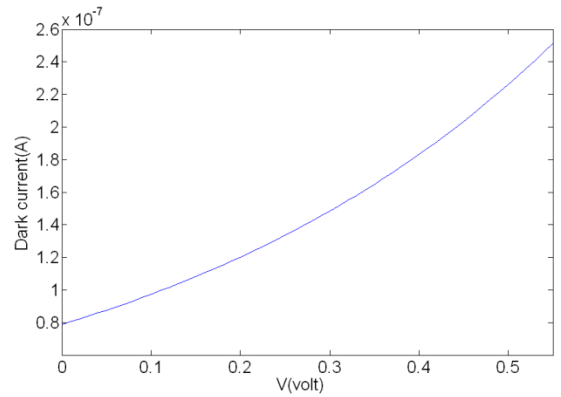
The behavior of dark current in (QRIPs) versus different values of bias voltages is shown in Fig. 2. The values of the parameters for QRIP are same as used in Ref [1]. These curves prove the semi exponential equation of dark current that we came to it in this paper. In conclusion we show in this paper that using a simple formula one can get an easier way in order to avoid dealing with complicated calculations in studying dark current characteristics of QRIPs. As we expect the operating temperature should be low (<50) to get low dark current and hence lower noise. By increasing the density of quantum wires  $\sum QR$  the dark current will increase.



**Fig. 2.** Dark current behavior for different biasing voltages for  $\sum QR = 1.5 \cdot 10^{10} \text{ cm}^{-2}$ ,  $T = 40$ ,  $a_{QR} = 10 \text{ nm}$ .



**Fig. 3.** Dark current versus biasing voltage for  $\sum QR = 10^{10} \text{ cm}^{-2}$ ,  $T = 40 \text{ K}$ ,  $a_{QR} = 15 \text{ nm}$ .



**Fig. 4.** Dark current versus different biasing voltages for  $\sum QR = 10^9 \text{ cm}^{-2}$ ,  $T = 50 \text{ K}$ ,  $a_{QR} = 10 \text{ nm}$ .

## V. CONCLUSION

In this article we have considered quantum wire infrared photodetector under dark current condition and also its physical model. A new simple mathematical procedure is given to find the dark current formula in QRIP. The analysis is carried out by considering important parameters of QRIP, such as the maximum number of electrons in quantum wires  $\langle N \rangle$ , their lateral characteristic size, and the nonuniformity factor. The results are in good agreement with the result of Ref.[1,2].

## REFERENCES

- [1] V. Ryzhii, I. Khmyrova, M. Ryzhii, and V. Mitin, "Comparison of dark current ,responsivity and detectivity in different intersubband infrared photodetectors," *Semicond.Sci.Technol.*, vol. 19, pp. 8-16, 2004.
- [2] A. Nasr, A. Aboshosha, and S. M. Al-Adl, "Dark current characteristics of Quantum wire infrared photodetectors," *IET Optoelectron.*, vol. 1, no. 3, pp.140-145, 2007.
- [3] M. S. El\_Tokhy, I. I. Mahmoud, and H. A. Komber, "Characteristics analysis of quantum wire infrared photodetectors under both dark and illumination conditions," *J.Infrared Physics and Tech.*, article in press, 2010.

- [4] V. Ryzhii, I. Khmyrova, M. Ryzhii, and M. Ershov, "Comparison studies of Infrared photodetectors with a Quantum-Well and a Quantum-Wire Base," *Journal de Physique IV*, vol. 6, 1996.
- [5] V. Ryzhii, "Characteristics of quantum well infrared photodetectors," *J. Appl. Phys.*, vol. 81, no. 9, pp.6442-6448, 1997.