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THz RESPONSE OF SPLIT-RING-RESONATOR BASED ON NONLINEAR METAMATERIALS

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ABSTRACT
In this paper, we outline a novel class of materials, called metamaterials, with negative refractive index and a high degree of nonlinearity. A brief summary is given on the basic theory of optics to show how this condition arrives for metamaterials to be designed into an antenna with split-ring-resonator (SRR). An example is given on the modeling of such SRR-based metamaterials. The goal of this paper is to explore whether SRR system, through engineering of their geometry (i.e. change of coupling) and permittivity, shows any sensitivity on the femtosecond response of quantum breathers by quantum calculations hitherto not done in metamaterials. This study is quite realistic to understand quantum localization by nonlinearity that is essential for many small-structured nano-devices. Therefore, this study should be viewed as working towards understanding of many nano structured devices.

Keywords: Metamaterial, SRR, Quantum Breathers, femtosecond response.

I. INTRODUCTION

With respect to a voluminous work done on classical breathers [1] on metamaterials which are theoretically and experimentally very important with a lot of interesting applications [2-4], not much work has been done on their quantum perspective. This brings us to the new and evolving field of quantum breathers that have already been studied in nonlinear optical materials with charge defect [5]. Before we go into some details in this field, let us briefly describe metamaterials: Substances with both negative dielectric constants (ε) and magnetic permeability (μ) possess a negative index of refraction and consequently they exhibit a variety of interesting optical properties. These negative indexed materials (NIM) called "metamaterials" (MM) do not occur in nature, and only recently it has been possible to artificially fabricate them as split-ring-resonators (SRRs) for antenna arrays, which are embedded in a nonlinear Kerr-medium so that the entire system can be treated as nonlinear. Based on a theory of Pendry et al [2], Smith et al [3] engineered a practical metamaterial with negative refraction property that can even cross the diffraction limit for various applications [4]. Next let us look at discrete breathers (DBs).

Discrete breathers (DBs) also known as intrinsic localized modes (ILM) are nonlinear excitations that are produced by the nonlinearity and discreteness of the periodic lattice [1,6]. They are formed as a self-consistent coupling between the mode and the system nonlinearity. Thus, DB modifies the local property that provides the environment for the DB to exist. The appropriate length scale drives us to nano-range in MM containing SRR elements. Thus, localization assumes more significance that arises due to interplay of nonlinearity and discreteness.

To characterize classical DBs, the bulk system was the right tool [7], but for very small (nano) systems, the laws of classical mechanics are not valid that brings us to the quantum breathers (QBs) [8,9]. These are observed in many interesting fields, viz. BEC in photonic lattices, antiferromagnetics, DNA, metamaterials (SRR), Josephson junctions, ferroelectrics, etc. (see Ref. [5,10] for all the relevant references)

The signature of QBs is manifested in two-phonon bound state. The interplay of nonlinearity and discreteness with respect to charge density in terms of both space and time domains gives rise to a K-G model through variational principle based on our discrete Hamiltonian that embodies both coupling and linear permittivity [1]. On the one hand, the 'geometry' of such engineered metamaterials that are modeled as R-L-C circuit can be varied in terms of slit size and other parameters to change the coupling of the system. This coupling may be a sensitive variable to the formation of quantum breathers and hence on their lifetime in femtoseconds that has important implications for THz applications. On the other hand, the linear permittivity that is present in our Hamiltonian of the system may also be considered to play a role in the femtosecond response of SRR based metamaterials for antenna applications.

II. SPLIT-RING-RESONATOR STRUCTURE

The structure of Single-Ring-Resonator based on "metamaterial" (MM), i.e., an artificial structure consisting of metallic wires responsible for the negative permittivity is shown in Fig. 1. The SRRs may have different shapes. The optical and electrical properties of a metamaterial can be harnessed by the proper use of SRRs that has application in antennas. However, very high accuracy is required for the fabrication of SRRs. Unlike natural materials, MMs also shows relatively large magnetic response at THz frequency and hence their THz application assumes more significance. Basically, in normal materials the wave vector and pointing vector are parallel, whereas in MMs they are anti-parallel

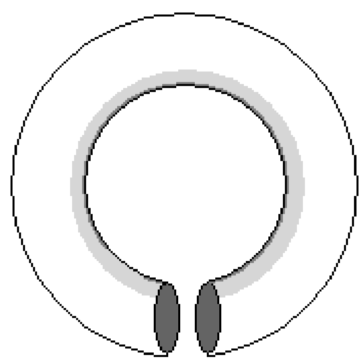

Fig. 1: A typical assembly of SRRs.

Let us consider a simple case, as shown in Fig. 2. This is a typical SRR assembly due to Kerr dielectric filling between the SRR slits with linear permittivity (ε_l). A one-dimensional discrete, periodic array of identical nonlinear

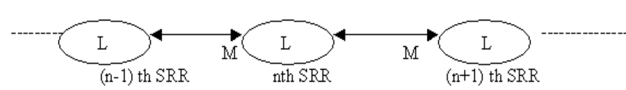

Fig. 2: Three adjacent split-ring resonators

SRRs are considered here for a device-oriented model for an antenna-array application having a varying inductance (L) and mutual inductance (M) values [1, 12].

III. THEORETICAL ASPECT

Quantum localization in K-G system has been studied by many researchers with four atom lattice, particularly by Proville [10]. Based on the discrete Hamiltonian [11], by using the 'variational principle', the governing equation in relation to the charge density in the SRR assembly can be expressed as nonlinear Klein-Gordon dynamical equation that is valid for metamaterials. This gives rise to dark and bright solitons, dark and bright discrete breathers as well as breather pulses [1]. The latter cannot be obtained by discrete nonlinear Schrodinger equation (NLSE). So, our approach on a Klein-Gordon lattice model is quite richer in the theoretical context as well as in the practical applications. Such characteristics of classical discrete breathers are mentioned here, as they also involve 'localization' due to nonlinearity that is our focus in the present work.

In this context, it is to be noted that our discrete Hamiltonian that was developed for another important nonlinear optical material, such as ferroelectrics, can also be used [6] for the purpose of second quantization by Bosonic operators to reveal the presence of quantum breathers. However, in the present work, for the temporal evolution spectra of quanta, we have adopted a generalized Hamiltonian that is quantized for our purpose. It is worth mentioning that Proville [10] took a 'four-atom' lattice for studying quantum breathers, but our method is capable of taking any number of quanta on an arbitrary number of sites in a non-periodic boundary condition.

The generalized Hamiltonian for the Klein-Gordon system for order parameter (y_n) at nth site can be expressed as:

$$H = \sum_n \frac{p_n^2}{2m} + \frac{A}{2} y_n^2 + \frac{B}{4} y_n^4 + k(y_n - y_{n-1})^2 \quad , \qquad (1)$$

the first term is momentum at nth site (p_n), the second and third terms are nonlinear potential and the last one includes a coupling term (λ); the third term includes both linear permittivity (ε_l)and focusing nonlinearity ($\alpha=+1$). Here A and B are two constants. From Eq. (1), we deduced the equation of motion after rescaling of time with the timescale as 3.048 fs for a coupling value of λ=0.01 for SRR based metamaterials.

Now, for second quantization, we need to make the creation and annihilation Bosonic operators that act on the above Hamiltonian. In order to characterize quantum breathers, let us resort to $\langle n_i \rangle (t) = \langle \psi_t | \hat{n}_i | \psi_t \rangle$ for the temporal evolution of quanta at each site of the system. We take i-th eigenstate of the Hamiltonian, and then make it time dependent:

$$|\Psi_i(t)\rangle = \sum_i b_i \exp(-iE_i t/\hbar) |\psi_i\rangle \qquad (2)$$

where Ψ_i and E_i are the i-th eigenvector and eigenvalue respectively, t is time, the Planck's constant (h) taken as unity and $b_i = \langle \psi_i | \Psi(0) \rangle$ for each site i and for a given range of t, where $\Psi(0)$ stands for initial state.

IV. RESULTS AND DISCUSSION

For the nonlinear Klein-Gordon lattice, in contrast with discrete nonlinear Schrodinger equation, the energy is not completely transferred between the anharmonic oscillators and there is a critical redistribution time of quanta [9] that is proportional to the quantum breathers' lifetime in femtoseconds (fs) which has an important implication in THz devices. Unlike natural materials, metamaterials show a large magnetic response at THz frequency and hence their THz applications make significant advances [13].

In a periodic boundary condition with a Bloch function, the two-phonon bound state has already been observed in ferroelectrics [5] that is a signature of quantum breathers. However, as

said earlier, here we are dealing with non-periodic boundary condition for the temporal evolution spectra of quanta by Eq. (2). A typical spectrum is shown in Fig. 2. This is for 12 quanta for a focusing nonlinearity $\alpha=+1$ and linear permittivity ε_l=0.2 that is less than that taken by Lazarides et al [12] for a coupling value of 0.01. Here, the initial localization is mainly at the first site and then there is a fast redistribution of quanta between the other two sites until they become equal or almost equal. Here, the critical time for redistribution (t_{re}) is around 1.2 that is proportional to quantum breathers' lifetime. This is about 3.6 fs (278 THz) for this value of coupling. This value seems to be on the very much lower side of that compared to ferroelectric materials, such as lithium niobate, with 12 quanta, i.e. 562 fs (about 2 THz) for a coupling of 0.1, but with a much higher nonlinearity.

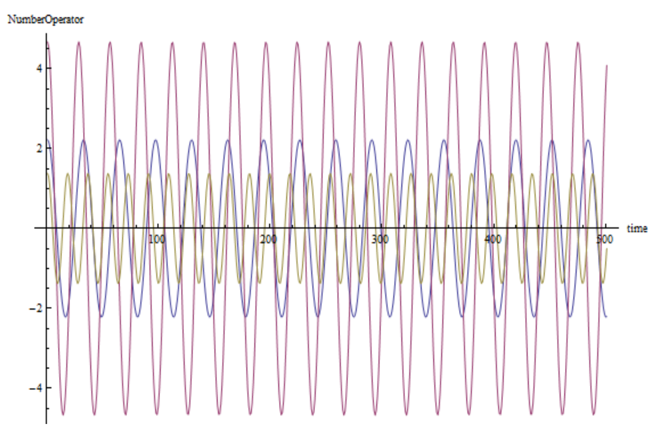

Fig. 3: Spectra for SRR based metamaterials.

The temporal evolution spectra are quite sensitive to both coupling and permittivity values that are used in our computation as input data. For a change of three orders of magnitude of permittivity, the lifetime changes from 16.4 to 17.2 THz that is not significant, but it is quite higher for lower number of quanta. However, for a change of coupling within the SRR assembly of more than two orders of magnitude, the lifetime changes from 62.5 to 157.5 THz (higher THz response with lower permittivity).

V. CONCLUSION

This study is realistic to understand quantum localization by nonlinearity that is essential for many small-structured nano-devices. Quantum localization in K-G system has been studied by many researchers with four atom lattice. Here, for quantization of our discrete Hamiltonian by Bosonic operators, we have taken arbitrary number of quanta on any number of sites to generate the temporal evolution spectra of quanta. A typical diagram is shown Fig. 3. The time of redistribution of quanta in femtoseconds shows interesting behaviour with permittivity from design viewpoint in THz applications.

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