DELAY METRIC FOR ON-CHIP RLCG COUPLED INTERCONNECTS FOR RAMP INPUT

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ABSTRACT
In this paper we have put forward an analytical model which could accurately compute the on chip interconnect delay using distributed RLCG segments. With the increasing level of on chip integration the interconnect delay has acquired prominence for performance driven layout synthesis. Also in higher frequency range of the order of GHz, the effect of shunt conductance and inductance cannot be ignored. Our proposed analytical model is based on II and III central moment of the interconnect transfer function. We have taken the ramp as input signal as for RLCG interconnection Elmore Delay can deviate up to 100% or more than the spice computed delay since it is independent of rise time. Experimental results show that the estimated delay using our first few central moments based model performed for 0.18μm technology 5% of SPICE computed delay across a wide range of interconnect parameter values.

Keywords: Delay Calculation; RLCG Interconnect; Central Moments; Ramp Input, Even and Odd Mode

I. INTRODUCTION
Transmission line modelling of on chip interconnects has received a great and increasing interest over the last couple of decades and is still a challenging problem. It is critical to model the transmission path when designing a high-performance, high-speed serial interconnection system. Simple but effective analytical delay models of interconnects are useful for IC designers, to avoid the timing issue problem and to optimize the design, such as minimizing delay [1-2] as the interconnect delay are playing more dominant role than the gate delay. Hence, it is necessary to derive accurate and effective delay estimation models for interconnects at higher frequency. Elmore delay model [1], which is simple in form and easy to be used, has been widely adopted to estimate the interconnect delays in local and global interconnects. But Elmore delay cannot be used for ramp input as it is independent of the rise time and it takes only the resistance and capacitance effects into account. The on-chip interconnect may be modelled either as lumped, distributed or as the full wave models depending on the frequency of operation and rise time of the input signal. At relatively lower frequency, interconnect may be modelled as distributed RC segments [3-6]. In order to capture the high frequency effect such as, undershoot, overshoot and ringing, the interconnect is modelled as a distributed RLC network [7-10] and the accuracy in performance estimation of the interconnect eventually got improved. Resistance (due to the skin effect) and conductance (due to dielectric absorption) are in fact dependent on the frequency. Both effects result in increased attenuation at higher frequencies thus the effect of G at higher frequency cannot be ignored in many practical situations especially in the very high frequency domain used in the present VLSI design [11]. It is necessary to use a model, which includes the effect of inductance and conductance. There are several approaches proposed to estimate the on-chip interconnect performance characteristic; where the interconnect is modelled as distributed RLCG segment [12-14]. In [15], the interconnect line is modelled as distributed RLCG elements and the frequency response is calculated and it is shown that RLCG consideration is suitable up to 110 GHz frequency of operation. Hu et al. [16] have proposed an interconnect RLCG state space model in time domain with computation complexity of O (N), where N is the total system order. An analytical delay model for distributed on-chip RLCG interconnects has been proposed in [12] taking step function as input. Another delay model proposed in [13] calculates the delay of distributed RLCG interconnects by taking into consideration the coupling effect by using difference model approach. In [14] also, the delay is calculated for on-chip global RLCG interconnect using step input, but to the best of our knowledge there is no such analytical explicit delay estimation models proposed which is based on the first few central moments for ramp input for distributed RLCG segments.

We have made the following contributions in this paper: We have proposed an analytical model for the delay estimation of the on-chip interconnect for different high frequency mode of operations considering the conductance (G) effect.

The rest of the paper is organized as follows: Section II discusses the basic theory, transmission line model, central moments of transfer function; Section III describes the proposed model for delay estimation for even and odd mode.
Section IV shows the simulation results. Finally Section V concludes the paper.

II. BASIC THEORY

For a simple input source terminated transmission line, we can write the transfer function as,

\[
H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{sC(R, \cosh(\alpha d) + Z_c \sinh(\alpha d)) + (R, / Z_c) \sinh(\alpha d) + \cosh(\alpha d)}
\]

where \( \gamma = \sqrt{(R + sL)(G + sC)} \) is the propagation constant and \( Z_c = \sqrt{(R + sL)(G + sC)} \) is the characteristic impedance of the line. \( R, L \) and \( C \) are the per-unit-length resistance, inductance, and capacitance of the transmission line, respectively. \( d \) is the length of the line. The series resistance is given by, \( R_s = R_d + R_{imp} \), where \( R_d \) is the driver resistance and \( R_{imp} \) the additional termination resistance. We assume that at the given frequency of interest, the dielectric loss and conductance values are negligibly small. The driver resistance is assumed to be linear. Now the RLCG interconnect can be considered as either lossless or lossy.

![Figure 1: Two-Port Model of a Distributed RLCG Line.](image)

II.1 Lossy Interconnect

For a lossy transmission line shown in Figure 1, the central moments are given by following equations:

\[
\begin{align*}
\mu_2 &= \alpha_1 + \beta_1 + \alpha_2 + \epsilon_1 + \phi_1 \\
\mu_3 &= \alpha_2 + \beta_1 + \alpha_3 + \epsilon_2 + \kappa_1 + \phi_1 \\
\end{align*}
\]

where \( \alpha_1 = R_s^z[-17C^2G - 9RC^2G - C^3G] \)

\( \beta_1 = R_s^z[-\frac{58}{3}RC^2G - 3C^2 - 2RC^2] \)

\( \chi = R_s^z\left[-\frac{4}{3}RC^2 - \frac{25}{3}CLG \right] \)

\( \epsilon_1 = -(LC + 3RLGC) \)

\( \phi_1 = \left(1 + \frac{9}{2}RG + 9R_sG \right) \)

\( \alpha_2 = R_s^z(182C^3G + 66RC^2G) \)

\( \beta_2 = R_s^z(214C^3RG + 14C^3 + C^2RG + 6RC^3) \)

\( X_2 = R_s^z(14RC^3 - 25C^2GL + 21RC^2LG) \)

\( \epsilon_2 = R_s^z(-3LC^2 - 37RC^2LG + \frac{3}{2}2RLC^2) \)

\( \kappa_2 = \left(\frac{5}{2}RLC^2 - 3L^2GC \right) \)

\( \phi_2 = \left(1 + \frac{17}{2}RG + 17R_sG \right) \)

R, L, C and G are the per-unit-length resistance, inductance, and capacitance and conductance of the transmission line, respectively.

II.2 Lossless Interconnect

For an unloaded lossless transmission line driven by a step input, it is well known that the optimal termination resistance is \( R_e = Z_0 \). With this termination, the ideal signal is the input step delayed by the time-of-flight along the line, and is given by, \( T_f = \sqrt{LC} \). The following discussion shows that this ideal response is indeed obtained when the central moments of the impulse response are minimized. For the lossless line in Figure 2, the transfer function is given by [17-18],

\[
H(s) = \frac{1}{(R, / Z_c) \sinh(\gamma d) + \cosh(\gamma d)}
\]

where \( \gamma \) and \( Z_0 \) are the propagation constant and the characteristic impedance, respectively and are defined as, \( \gamma = s\sqrt{LC} \) and \( Z_c = s\sqrt{LC} \). For this transfer function, the second and third central moments of the impulse response are symbolically given as:

\[
\mu_2 = -CLd^2 - 3R_sC^2d^2 \]

\[
\mu_3 = -3R_sC^2Ld^3 + 14R_sC^4d^3 \]

The above equations can be obtained by putting \( R = 0 \) and \( G = 0 \) in (2).

![Figure 2: Lossless Transmission Line.](image)

Solving for \( \mu_2 = 0 \) from (4) yields \( i\sqrt{L/3C} \) and \( -i\sqrt{L/3C} \) as roots. Again solving \( \mu_1 = 0 \) from (4) yields \( \sqrt{3L/14C} \) and \( -\sqrt{3L/14C} \) as roots.

The positive root provides the solution \( R_e = Z_0 \), approximately. Then, the transfer function given as,

\[
H(s) = \frac{1}{\sinh(\gamma d) + \cosh(\gamma d)} = e^{-\gamma d}
\]
where $T_f = \sqrt{L/C}d$ is the time of flight. Then it is easy to show that this transfer function provides the desired ideal waveform at the output of the transmission line,

$$v_o(t) = v_i(t - T_f)$$  \hspace{1cm} (6)

From (6), it can be inferred that the ideal impulse response for a lossless transmission line is symmetric and localized (zero dispersion) about its mean, $\mu = \sqrt{LC}d$. Conversely, forcing the impulse response to be symmetric and localized about the mean ensures critical damping.

So from (6) we can write the following equation:

$$V_o(s) = V_i(s)e^{-sT_f}$$  \hspace{1cm} (7)

In case of ramp input,

$$V_i(s) = \frac{V_{io}}{s^2}$$  \hspace{1cm} (8)

Substituting (8) in (7) we get,

$$V_o(s) = \frac{V_{io}}{s}e^{-sT_f}$$  \hspace{1cm} (9)

Taking inverse Laplace transform of (9),

$$V_o(t) = V_{io}(t - T_f) u(t - T_f)$$  \hspace{1cm} (10)

In order to calculate the time delay we take $V_{o}(s) = 0.5V_{io}$ at time $t = T_D$ and hence, substituting in (10), we have,

$$0.5V_{io} = V_{io}(T_D - T_f) u(t - T_f)$$  \hspace{1cm} (11)

So, for $t \geq T_f$ the $T_D$ is given as,

$$T_D = T_f + 0.5$$  \hspace{1cm} (12)

The above equation (12) is the proposed closed form expression for delay for lossless transmission line RLCG interconnects system.

### III. PROPOSED DELAY MODEL

#### III.1 Calculation of Delay for Even Mode

In order to calculate the exact time delay in two parallel RLCG line, we consider the mutual inductance between two inductors as $M$. Figure 3 shows two highly coupled transmission line system.

![Highly Coupled Transmission Line](image)

From (1) and for a simple input source terminated transmission line, we can write the transfer function as,

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{sC(M/Z_o)\sinh(\gamma_d) + \cosh(\gamma_d)}$$  \hspace{1cm} (13)

where $\gamma = \sqrt{(R + s(L + M))(G + sC)}$ is the propagation constant and $Z_o = \sqrt{(R + s(L + M))(G + sC)}$ is the characteristic impedance for even mode of the line. R, L, M, C and G are the per-unit-length resistance, inductance, mutual inductance, capacitance and conductance parameters of the transmission line, respectively; $d$ is the length of the line. The series resistance is given by $R = R_d + R_e$; where $R_d$ is the driver resistance and $R_e$ the termination resistance. We assume that the dielectric loss and hence the conductance, G to be negligibly small. The driver resistance is assumed to be linear. For an unloaded lossless transmission line driven by a step input, it is well known that the optimal termination resistance is $R_e = Z_o$. With this termination, the ideal signal is the input step delayed by the time-of-flight along the line, is given by $T_f = \sqrt{(L + M)C}d$.

The following discussion shows that this ideal response is indeed obtained when the central moments of the impulse response are minimized. For the lossless line as shown in Figure 2, the transfer function is given by,

$$H(s) = \frac{1}{(R_e/Z_o)\sinh(\gamma_d) + \cosh(\gamma_d)}$$  \hspace{1cm} (14)

where $\gamma = \sqrt{(R + s(L + M))(G + sC)}$ and $Z_o = \sqrt{(L + M)/C}$. For this transfer function, the second and third central moments of the impulse response are symbolically given as:

$$\mu_2 = -C(L + M)d^2 - 3R_e^2C_1^2d^2$$

$$\mu_3 = -3R_eC_1^2(L + M)d^3 + 14R_e^2C_1^3d^3$$  \hspace{1cm} (15)

Solving for $\mu_2 = 0$ from equation (15) yields $i\sqrt{(L + M)/3C}$ and $-i\sqrt{(L + M)/3C}$ as roots. Again solving $\mu_3 = 0$ from equation (15) yields $0, \sqrt{3(L + M)/14C}$ and $-\sqrt{3(L + M)/14C}$ as roots. The positive root provides the solution as $R_e = Z_o$, approximately. Then, the transfer function is given as,

$$H(s) = \frac{1}{\sinh(\gamma_d) + \cosh(\gamma_d)} = e^{-sT_f}$$  \hspace{1cm} (16)

where, $T_f = \sqrt{(L + M)C}d$ is the time of flight. It can be shown that this transfer function provides the desired ideal waveform at the output of the transmission line.

$$v_o(t) = v_i(t - T_f)$$  \hspace{1cm} (17)

From (17), it can be inferred that the ideal impulse response for a lossless transmission line is symmetric and localized (zero dispersion) about its mean, $\mu = \sqrt{(L + M)C}d$. Conversely, forcing the impulse response to be symmetric and localized about the mean ensures critical damping.

So from (17), we can write the following equation:

$$V_o(s) = V_i(s)e^{-sT_f}$$  \hspace{1cm} (18)
In case of ramp input, 
\[ V_r(s) = \frac{V_{DD}}{s^2} \]  
(19)
Substituting (19) in (18) we get, 
\[ V_r(s) = \frac{V_{DD}}{s} e^{-sT_r} \]  
(20)
Taking inverse Laplace transform of (20) yields, 
\[ V_r(t) = V_{DD}(t - T_{je})u(t - T_{je}) \]  
(21)
For the calculation of the time delay, we take \( V_0(s) = 0.5V_{DD} \) at time \( t = T_0 \) and hence substituting in (21), we have 
\[ 0.5V_{DD} = V_{DD}(T_D - T_{je})u(t - T_{je}) \]  
(22)
So for \( t \geq T_{je} \), \( T_D \) is given as 
\[ T_D = T_{je} + 0.5 \]  
(23)

The above equation (23) is the proposed closed form expression for delay for lossless transmission line RLCG tree circuit in even mode and with mutual inductance.

### III 2 Calculation of the Delay in Odd Mode

Again from (1) for a simple input source terminated transmission line, we can write the transfer function as, 
\[ H(s) = \frac{V_o(s)}{V_i(s)} \]  
(24)

where, 
\[ \gamma_o = \sqrt{\frac{|R + sL - M|}{G + sC}} \]  
and 
\[ Z_m = \sqrt{\frac{(R + sL - M)}{(G + sC)}} \]  
The above equation (24) is the proposed closed form expression for delay for lossless transmission line RLCG tree circuit in odd mode and with mutual inductance.

### IV. EXPERIMENTAL RESULTS

The proposed model has been tested by comparing its results with the SPICE results. The configuration of circuit for simulation is shown in Figure 3. The high-speed interconnect system consists of two coupled interconnect lines and ground and the length of the lines is \( d = 10 \) mm. The sample dimensions of the cross sections of a minimum sized wire in a 0.18\( \mu \)m technology are given in Figure 4.
Table I. Extracted values of R, L, C, G

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Value/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance(R)</td>
<td>120 kΩ/m</td>
</tr>
<tr>
<td>Inductance(L)</td>
<td>270 nH/m</td>
</tr>
<tr>
<td>Conductance(G)</td>
<td>15 pS/m</td>
</tr>
<tr>
<td>Capacitance(C)</td>
<td>240 pF/m</td>
</tr>
<tr>
<td>Mutual Inductance(M)</td>
<td>54 nH/m</td>
</tr>
</tbody>
</table>

In the case of very high frequencies as in GHz scale, the inductive effect starts to play a crucial role and it should be included for complete delay analysis. The configuration of circuit for simulation is shown in Figure 5.

For each RLCG network source we put a driver, where the driver is a ramp voltage source followed by a resistor. The results are based on (23) and (33) for 0.18 µm process. The left end of the first line of Figure 5 is excited by a 1V ramp form voltage with rise/fall times 0.5 ns and a pulse width of 1ns. In Table II, the 50% delay for even mode and the Elmore delay is compared for various values of the driver resistance R, and the load capacitance C_l when the length of the RLCG interconnect is kept constant. In the similar way, in Table III the 50 % delay for odd mode and the Elmore delay are compared. In Table IV and table V, comparative results of our proposed delay model with the SPICE delay are given in the similar way as we did for the comparison of our proposed model and Elmore delay model discussed above as in Table II and Table III.

V. CONCLUSIONS

In this paper we have proposed an accurate delay analysis approach for distributed RLCG interconnect line under ramp input. The use of transmission line model in our study gives a very accurate estimate of the actual delay. We derived the transient response in time domain function of ramp input. We can see that when inductance is taken into consideration, the Elmore approach could lead to an error of average 10% compared to the actual 50% delay calculated using our approach.

Table II. Experimental result under Ramp input for even mode

<table>
<thead>
<tr>
<th>Ex.</th>
<th>R_e(Ω)</th>
<th>C_L(fF)</th>
<th>L(µm)</th>
<th>T_delay(ps)</th>
<th>Proposed Delay Model (ps)</th>
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<tr>
<td>4</td>
<td>10</td>
<td>750</td>
<td>100</td>
<td>0.9310</td>
<td>0.9453</td>
</tr>
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<td>100</td>
<td>0.7920</td>
<td>0.7875</td>
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<td>6</td>
<td>100</td>
<td>1500</td>
<td>100</td>
<td>0.9895</td>
<td>1.0872</td>
</tr>
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</table>

Table III. Experimental result under Ramp input for odd mode

<table>
<thead>
<tr>
<th>Ex.</th>
<th>R_e(Ω)</th>
<th>C_L(fF)</th>
<th>L(µm)</th>
<th>T_delay(ps)</th>
<th>Proposed Delay Model (ps)</th>
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Table IV. Comparative result with spice under Ramp input for even mode

<table>
<thead>
<tr>
<th>Ex.</th>
<th>R_e(Ω)</th>
<th>C_L(fF)</th>
<th>L(µm)</th>
<th>Delay Model (ps)</th>
<th>SPICE (ps)</th>
<th>% Error</th>
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Table V. Comparative result with spice under Ramp input for odd mode

<table>
<thead>
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<th>Ex.</th>
<th>R_e(Ω)</th>
<th>C_L(fF)</th>
<th>L(µm)</th>
<th>Delay Model (ps)</th>
<th>SPICE (ps)</th>
<th>% Error</th>
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References


